A Two-stage ascending auction protocol for digital goods *

Toyotaka Sakai[†], Takuya Goto[‡], and Yuya Ishikawa[§]

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Abstract

We design a two-stage protocol for auctioning multiple, homogeneous digital goods with no marginal cost. The difficulty of designing protocols for such goods lies in determining the number of goods to be auctioned. In the first stage of our protocol, every bidder submits bids, and half of the number of bids is set as the number of auctioned goods. In the second stage, a uniform price, ascending auction protocol is conducted. We explain the intent of this design and report the results of our real use.

Keywords: Ascending auction, Uniform pricing, Digital goods, Blockchain, Non-fungible token.

1 Introduction

In the traditional practice of auctions, the number of auctioned goods has been exogenously given. Such a number is naturally determined by resource constrains. For example, artwork such as paintings or statues uniquely exists, and the amount of treasury bonds is determined by the issuing government. However, this is not the case with digital goods. A seller can produce any number of digital goods at virtually no marginal cost. This freedom raises a difficult question: how many of the goods are to be produced? In an ideal case with perfect information, a monopolistic seller can select a quantity q^* that maximizes revenue $q \cdot P(q)$, where $P(q)$ denotes the market price at the total supply q. However, the form of P is unknown when the goods are new items in a marketplace. We, in Gaudiy Inc., were faced with this design problem when we constructed a primary market for blockchain game cards. The cards are non-fungible tokens (NFTs) with no material form. They can be produced at zero marginal cost.

In the literature of algorithmic mechanism design, a random sampling approach has been proposed for auctioning digital goods (Goldberg and Hartline 2001). Its procedure consists of dividing a set of bidders into two groups, implementing the Vickrey auction in each of the groups, and exchanging two Vickrey prices from one group to another.[1] We did not employ this

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†Corresponding author: Department of Economics, Keio University. `tsakai@keio.jp`

‡Gaudiy Inc.

§Gaudiy Inc.

[1] An obtained Vickrey price in Group A is used in Group B while an obtained Vickrey price in

approach because of three concerns. First, the Vickrey auction often yields low revenue in multi-item auctions (Engelbrecht-Wiggans and Kahn 1998). Second, our participants appear not to prefer the idea of exchanging two prices. Third, the law of one price is not respected because bidders are separated. Blockchain game cards are often sold in a secondary market, and the multiplicity of prices in our primary market makes the finding of a reasonable price in a secondary market difficult. In summary, we needed an auction protocol that gives everyone the same price and yields a satisfactory amount of revenue.

We developed a novel, two-stage ascending auction protocol with endogenous determination of a supply amount. We used it for selling digital cards in an auction that was held for seven days. Any participant can submit at most two bids and raise the bids any time. At the end of the sixth day, *half* of the number of the submitted bids, say Q, is determined to be the number of the auctioned goods. On the seventh day, participants seriously start raising their bids. At the end of this day, the auction ends; Q bids from above win goods, and $Q+1$-th highest bid becomes the uniform price. Any winning bid that was initially submitted in the first six days gets a 10% discount coupon.[2] The coupon is to encourage participants to submit initial bids in the first six days. We term this protocol *Gaudiy-Sakai two-stage protocol*.

Group B is used in Group A. By doing so, the amount of auctioned items is endogenously determined in the auction.

[2]This coupon is valid if an initial bid is updated at any point in the remaining auction period.

2 Model

2.1 Basic definitions

Let I be the set of participants.[3] Any participant $i \in I$ is associated with a two-dimensional valuation vector $v_i = (v_{i1}, v_{i2}) \in \mathbb{R}^2_+$, which is private information. When i wins no good with no payment, his utility is zero; when he wins one good with payment m, his utility is $v_{i1} - m$; and when i wins two goods with payment m, his utility is $v_{i1} + v_{i2} - m$. A participant $i \in I$ is said to have a *unit demand* if $v_{i2} = 0$.

Let $v = (v_i)_{i \in I}$ be a profile of valuation vectors. Because our auction is held for seven days, the set of times is given by $T = [0, 7]$. A bid vector of $i \in I$ at time $t \in T$ is

$$b^t_i = (b^t_{i1}, b^t_{i2}) \in \mathbb{R}^2_+, \tag{1}$$

and we assume that $b^0_{i1} = b^0_{i2} = 0$ and $b^t_{i1} \geq b^t_{i2}$ for all $t \in T$. Let $b^t = (b_i)^t_{i \in I}$ be a profile of bid vectors at t. Because we do not allow withdrawal, any b^t_{ik} is weakly increasing with respect to t. For any integer Z, $b^t[Z]$ denotes the Z-th highest value among all $2 \cdot |I|$ bids at t.

2.2 Protocol

Any participant can submit and raise at most two bids during the auction period of seven days.

First stage. This consists of the first six days. Any participant can submit at most two bids and raise bids anytime. The minimum amount of bids is set at 500 yen, and a bid b^t_{ik} is *valid* if $b^t_{ik} \geq 500$. Let

[3]In our practice, any participant is identified with an account, which requires an SMS identification.

$b^6 = (b^6_{i1}, b^6_{i2})_{i\in I}$ be the profile of bid vectors at the end of the sixth day. The number of valid bids at b^6 is

$$|\{(i,k) \in I \times \{1,2\} : b^6_{ik} \geq 500\}|, \quad (2)$$

and its half, rounded up when it is odd, is

$$Q(b^6) = \lceil \frac{|\{(i,k) \in I \times \{1,2\} : b^6_{ik} \geq 500\}|}{2} \rceil. \quad (3)$$

Additionally, an upper bound M is imposed by the publisher of the game to keep the blockchain game of the cards balanced. The number of the auctioned cards is set to be

$$\bar{Q}(b^6, M) = \min\{Q(b^6), M\}. \quad (4)$$

The number $\bar{Q}(b^6, M)$ is announced to the participants through their computer display. For notational simplicity, when there is no danger of confusion, we denote $\bar{Q}(b^6, M)$ by Q.

Second stage. This only includes the seventh day. Participants who have already submitted bids at the first stage can raise their bids. New participants can also submit and raise bids. At any time $t \geq 6$, the *temporary price* at t is

$$p(b^t) = b^t[Q+1]. \quad (5)$$

If a participant wishes to raise his bid on this day, his bid must be equal or greater than $p(b^t)$ plus 500, i.e., 500 yen is the minimum incrementation amount.

At the end of the seventh day, the auction ends; $Q(b^6)$ bids from above become the winning bids and the uniform, the *execution price* is set as $Q+1$-th bid; that is,

$$p(b^7) = b^7[Q+1], \quad (6)$$

where $b^7[Q+1]$ denotes the $Q+1$-th highest bid, which is the highest losing bid.

Any winning bid that was initially submitted at the first stage gets a 10% discount coupon. For example, if a participant wins two goods, with one bid initially submitted at the first stage, with the other bid not, he has to pay

$$0.9 \cdot p(b^7) + p(b^7) = 1.9 \cdot p(b^7). \quad (7)$$

For any $i \in I$, b^7_i is said to be *optimal* at the second stage if for every $\tilde{b}^7_i$ and every $b^7_{-i} = (b^7_j)_{j\neq i}$, his utility from obtained goods minus payment at (b^7_i, b^7_{-i}) is weakly greater than that at $(\tilde{b}^7_i, b^7_{-i})$.

Before the auction, we explained all these rules to potential participants via a text website and video. To the best of our understanding, there was no confusion about the rules.

3 Properties

We summarize the intent of our auction design.

Fairness. Because our protocol satisfies the law of one price, it respects the principle of fairness that every two winners who purchase the same amount of goods pay the same amount of money. The uniqueness of the price in the primary market is useful in the secondary market where traders need to find a reasonable price.

Incentives. Incentive compatibility is not fully satisfied. This is because our "next-price" format is not incentive compatible for a multi-demand bidder (Engelbrecht-Wiggans and Kahn 1998).

Let us say that $(b^t_i)_{t\in T}$ is *constant* if $b^0_i = b^7_i$. The following example shows that truth-telling is not always profitable to participants.

Example 1 (**Truth-telling is not always profitable**). Let $I = \{1,2,3,4\}$, $v_1 = (10,8)$, $v_2 = (10,2)$, $v_3 = (7,0)$, and $v_4 = (2,0)$. Suppose that every participant takes a truth-telling, constant bid. Then three goods are determined to be auctioned at the end of the first stage. At the end of the second stage, participant 1 obtains two goods, pays $7+7$, and his utility is $10+8-(7+7)=4$.

Consider participant 1's constant bid $(\tilde{b}_1^t)_{t\in T}$ with $\tilde{b}_1^t = (10,1)$. If any $j \neq 1$ takes his truth-telling, constant bid and participant 1 takes $(\tilde{b}_1^t)_{t\in T}$, then three cards are auctioned, the uniform price is $p = 2$, and participant 1 obtains one good. Then participant 1's utility is 10-2=8, which is higher than 4. Therefore this deviation is profitable to him. □

However, our protocol satisfies certain nice properties on incentives. For example, it is more beneficial for winners if his bids were submitted at the first stage because of the coupon. Furthermore, our protocol is fully incentive compatible for a unit demand participant at the second stage; that is, truth-telling is his dominant strategy. This property is an immediate consequence of the fact that for such a bidder, our protocol at the second stage is essentially equivalent to the Vickrey auction. A non-essential difference from the Vickrey auction is that a dominant strategy is $\frac{10}{9}v_{i1}$, not v_{i1} itself, because of the coupon. This, in turn, ensures that the seller does not lose revenue by giving coupons. We summarize these facts on incentives in the following propositions.

Proposition 1 (**Optimal bids at the second stage**). *Consider any unit demand participant i. If i has submitted a bid at the first stage, then $b^7 = (\frac{10}{9}v_{i1}, 0)$ is his optimal bid at the second stage. Otherwise, $b^7 = (v_{i1}, 0)$ is his optimal bid at the second stage.*

Proof. It is well known that our dynamic, uniform price auction for multiple goods can be identified with the static, next price auction (see, for example Krishna 2009). Under the next price auction, truth-telling is a dominant strategy of any unit demand participant i. Therefore if i who has not submitted a bid at the first stage, his optimal bid at the second stage is $b^7 = (v_{i1}, 0)$. If i has submitted one bid at the first stage, then his utility from the first good minus payment is $v_{i1}^7 - 0.9p$, which is positive as long as $p > \frac{10}{9}v_{i1}$. Noting this fact and applying the standard argument on strategy-proofness of the second price auction, one can easily verify that $(\frac{10}{9}v_{i1}, 0)$ is i's optimal bid at the second stage. □

Proposition 2 (**Incentives to bid at the first stage**). *(i) If a participant wins goods, then he is more beneficial in the case that his winning bids were initially submitted at the first stage than in the case that the bids are not so; (ii) If $\frac{10}{9}v_{i1} > v_{j1} > v_{i1}$, then it is possible that i, j take their optimal bids at the second stage, i wins, but j loses. This "reversal" occurs when i submits his initial bid at the first stage, but j does not.*

Proof. The first part is trivial. Let us prove the second part. Suppose that $\frac{10}{9}v_{i1} > v_{j1} > v_{i1}$. Consider the case that i submits a initial bid at the first stage, but j does not. Let $b_{i1}^7 = \frac{10}{9}v_{i1}$ and $b_{j1}^7 = v_{j1}$. By Proposition 1, these bids are optimal bids at the second stage for i, j. Let $(b_h^7)_{h\neq i,j}$ be such that the highest losing bid p at $b^7 = (b_i^7, b_j^7, (b_h^7)_{h\neq i,j})$ satisfies $\frac{10}{9}v_{i1} > p > v_{j1}$. Then i wins a good but j does not. □

Smooth ending. Under many protocols of ascending auctions, a considerable number of last-minute pushes were observed in bidding. Such congestion hurts revenue and efficiency. To avoid it, some devices such as the activity rule and the 10-minutes rule are often employed (Milgrom 2004; Roth and Ockenfels 2002). In particular, we were concerned about programmed "bot" bidders who endlessly raise bids by a small incrementation. However, under our next-price design, participants do not have to do such bidding, because bidding high does not imply paying high; participants only need to pay the highest losing bid. Therefore we need not to employ the activity rule nor the ten-minutes rule, which simplified coding and eliminated any trouble caused by additional ending rules.

4 Results

The auction was held from March 6 to 13, 2020. The minimum bid is set to be 500 yen. When a participant wishes to update his bid and the temporary price is p, the updated bid must be at least $p+500$; that is, an incrementation of 500 yen is required to accelerate an ascending process.

First stage. 258 bids were submitted by 148 participants, and the upper bound was set to be $M = 150$. Therefore

$$Q(b^6) = \lceil \frac{258}{2} \rceil = 129, \quad (8)$$

$$\bar{Q}(b^6, M) = \min\{129, 150\} = 129, \quad (9)$$

$$p(b^6) = 2,398. \quad (10)$$

The distribution of the initial bids in seven days is given in Table 1:

Day	Bids
1st	113
2nd	31
3rd	36
4th	26
5th	25
6th	28
First stage	258
7th	13
Total	271

Table 1: The number of new bids at each day

There was no last-minute rush at the end of the first stage. The final update was done about seven minutes before the end of the first stage.

Second stage. Only 13 new bids are submitted by 10 participants. Therefore, throughout the auction process, 258 of the 271 bids, 148 of 158 participants, entered at the first stage. The ratios of new bids and participants exceed 95% and 93%, respectively. Hence we consider that the 10% discount coupon incentivised participants to submit bids at the first stage.

The execution price, which is the 130th highest of all bids, is $p(b^7) = 9,000$ yen. Since the starting price on the seventh day is $p(b^6) = 2,398$ yen, stringent competition occurred. Finally, 129 cards are sold to 76 winners, which means that winners obtained $\frac{129}{76} \approx 1.7$ cards on average.[4] We have observed that sufficiently many final bids exist around 9,000, which means that demand reduction did not occur in our auction.[5] In fact, as long as there are $Q+1$ high final bids, demand reduction

[4] Demand reduction is a phenomenon where many participants underbid the second good, which yields an unacceptably low price. See Ausubel et. al. (2014).

[5] 12 final bids are distributed from 8,000 to 9,000.

does not occur, or at least it does not hurt prices. Because of this, our limitation of the supply amount contributed in avoiding demand reduction.

The following example provides a situation where the supply amount is not limited in the sense that the number of participants is equal to the number of auctioned goods, and demand reduction occurs.

Example 2. (**Demand reduction when there are many goods**) Let $I = \{1, 2, 3, 4\}$, $v_1 = (10, 10)$, $v_2 = (10, 1)$, $v_3 = (10, 1)$, and $v_4 = (9, 1)$, where five "10, 9" are high valuations, and each of the four participants has at least one high valuation.

Consider the profile of constant bids $((b_i^t)_{t\in T})_{i\in I}$ such that $b_1^t = (10, 1), b_2^t = (10, 1), b_3^t = (10, 1)$, and $b_4^t = (9, 1)$. At this profile, $Q = 4$ cards are to be auctioned, and everyone obtains one card and pays $p = 1$. Therefore this is a situation of demand reduction. We shall show that $(b_i^7)_{i\in I}$, which yields this demand reduction, is a "low-price" Nash equilibrium at $t = 7$.

In this stuatin, everyone obtains a positive utility, so that no one has an incentive to change his bid to any other bid that makes him obtain no good. If a participant i changes his bid but still can obtain one good, then the uniform price continues to be $p = 1$, so that this change is not profitable to i. Furthermore, if a participant i changes his bid and obtains two goods, the price will be $p = 9$, which decreases his utility. Therefore no profitable deviation is possible to everyone. □

The average winning bid is 13,596 yen.[6] The seller's revenue is 9,000 yen × 129=1,161,000 yen. Although we do not have an evidence that this amount of revenue is fully maximized, it is fairly satisfactory and much more than expected. We consider that competitiveness at the second stage was the source of this success, which is realized by the limitation of the supply amount.

We are highly interested in understanding whether there was any last-minute rush. The auction closed on 22:00 on the seventh day, and the distribution of the number of updating bids in the last 20 minutes is summarized in Table 2.

Time	Updates	Time	Updates
21:40	8	21:50	9
21:41	2	21:51	5
21:42	1	21:52	3
21:43	2	21:53	6
21:44	3	21:54	5
21:45	3	21:55	4
21:46	4	21:56	3
21:47	7	21:57	7
21:48	7	21:58	7
21:49	3	21:59	5

Table 2: Number of bid updates at each time of the last 20 minutes of the seventh day

No last-minute rush was observed. We have statistically examined if there is any significant difference between the number of raising bids from 21:40 to 21:49 and that from 21:50 to 21:59. The Wilcoxon rank sum test does *not* reject the hypothesis that the two groups are from the same distribution with 5%.

[6] We drop one outlier value of 347,904 yen in this calculation.

5 Conclusion

We have designed and implemented a new auction protocol for digital goods. The main feature of the protocol is a two-stage design. The first stage is for an endogenous determination of the supply amount, and the second stage is for competition. The observed competitiveness seems to be the consequence of our way of controlling the supply amount, and we are unsure of other good alternatives. Virtues of this protocol include the respect of the law of one price, certain incentive properties, and absence of last-minute rush. Gaudiy Inc. plans to continue using this auction protocol and believes that it will be one of the standard protocols for auctioning digital goods. Further reports on our real use will be presented in future.

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